

136
Wgs

DESIGN OF A CONCRETE ARCH

BY

OSCAR WILLIAM RUDOLPH WANDERER

THESIS

FOR THE

DEGREE OF

BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

IN THE

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

1911

11

W18

UNIVERSITY OF ILLINOIS

May 25, 1911

This is to certify that the thesis prepared under my personal supervision by OSCAR WILLIAM RUDOLPH WANDERER entitled Design of a Concrete Arch is approved by me as meeting this part of the requirements for the degree of Bachelor of Science in Civil Engineering.

Ira O. Baker

Head of Department of Civil Engineering.

TABLE OF CONTENTS.

Introduction.....	Page	1.
Location.....		1.
Waterway.....		1.
Map of Bridge Site.....		2.
Specifications.....		3.
Design.		
Thickness of Crown.....		7.
Thickness of Abutment.....		7.
Table No. 1.		
Data for making $\Delta S + I$ constant.....		9
Table No. 2.		
Data for True Equilibrium Polygon...		10.
Table No. 3.		
Dead and Live Load Stresses.....		12.
Table No. 4.		
Temperature Stresses.....		13.
Table No. 5.		
Combined Stresses.....		15.
Drawings.		
$\Delta S + I = \text{constant}$		16.
Equilibrium Polygon.....		17.
Design of Bridge.....		18.

INTRODUCTION.

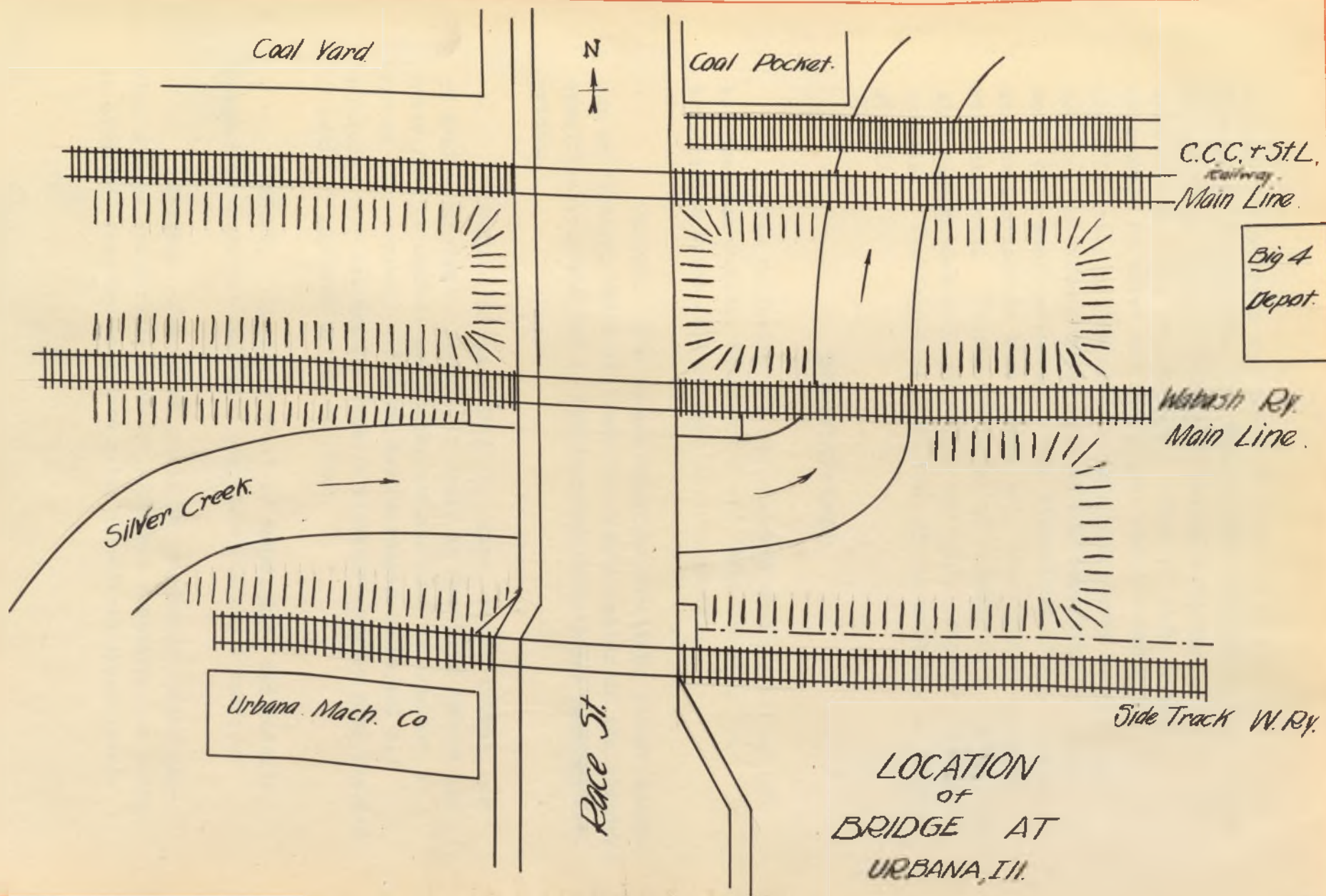
Since the author is very much interested in concrete work, particularly concrete arches, he has chosen for his thesis the design of a concrete arch to carry Race Street, Uroana over Silver Creek, commonly known as the "Boneyard". A concrete girder or a metal girder could be used; but the author considers that a concrete arch gives a better appearance than a concrete girder, besides probably being cheaper. A concrete structure is more durable than a metal girder, and if the maintenance cost of the latter be included the total cost of the former will probably not be much if any greater. Race Street is used chiefly by vehicles going to and from Crystal Lake Park, and therefore some attention to the appearance of the structure is required; and by the use of a little ornamentation the concrete arch can be made more beautiful than either the concrete girder or the metal bridge. The railway stations are only one block away, and therefore the bridge should be of a type that can be made as pleasing as possible.

LOCATION.

The creek flows along the southern embankment of the Wabash Railway, and then crosses the tracks about twenty feet to the east of the east side of Race Street. Page 2 is a map of the site. To prevent scour the railway company has erected a masonry retaining wall about fifty feet long and fourteen feet high. This retaining wall is at present in a very poor condition, large cracks showing and parts of the masonry being chipped off. The town has erected an "I" beam bridge which has a middle support. This support is undesirable as it obstructs the flow of water and allows the debris to catch.

WATERWAY.

Ample waterway must be provided, since in the flood season the creek rises five feet and reaches a width of twenty-five feet. The creek is also a good collector of debris, much silt and large



branches from trees and other objects being carried along. Further up the stream, the water flows through an arch under Main Street, Urbana, having a nominal waterway of about one hundred and fifty square feet. Part of this waterway is obstructed by water and gas pipes, which materially reduces the efficiency of the arch. This arch seems to give good service, and therefore an arch of the same size or larger would give ample waterway. The dimensions of the arch which will be designed in this thesis are span thirty feet, and rise six feet. Thirty-five feet of roadway with a five-foot sidewalk on each side will give the space necessary for the traffic. The height of the crown of the arch may be fixed at will.

SPECIFICATIONS.

The following are the standard specifications of the American Railway Engineering and Maintenance of Way Association which will be used in this design.

CEMENT. The cement shall be portland, either American or foreign, which will meet the requirements of the standard specifications adopted by the American Society for Testing Materials.

SAND. The sand shall be clean, sharp, coarse, and of grains varying in size. It shall be free from sticks and other foreign matter, but it may contain clay or loam not to exceed five percent. Crusher dust screened to reject all particles over one quarter inch in diameter may be used instead of sand, if approved by the engineer.

STONE. The stone shall be sound, hard and durable, crushed to sizes not to exceed two inches in any direction.

GRAVEL. The gravel shall be composed of clean pebbles of hard and durable stone of sizes not exceeding two inches in diameter, free from clay and other impurities except sand.

WATER. The water shall be clean and reasonably clear, free from sulphuric acid or strong alkalies.

MIXING BY MACHINE. A machine mixer shall be used wherever the volume of work will justify the expense of installing the plant. The necessary requirements for the machine shall be that a precise and regular proportioning of materials can be controlled, and the product as delivered shall be of the required consistency and be thoroughly mixed.

CONSISTENCY. The concrete shall be of such consistency that when dumped in place it will not require much tamping. It shall be spaded down, and be tamped sufficiently to level it off, after which the water should rise freely to the surface.

FORMS. Forms shall be well built, substantial and unyielding, properly braced or tied together by means of wire or rods, and shall conform to the lines given.

For all important work, the lumber used for face work shall be dressed on one side and both edges, and shall be sound and free from loose knots, secured to the studding or uprights in horizontal lines.

For backing and other rough work, undressed lumber may be used.

Where corners of the masonry and other projections liable to injury occur, suitable mouldings shall be placed in the angles of the forms to round or bevel them off.

Lumber once used in forms shall be cleaned before being used again.

The forms must not be removed within thirty-six hours after all the concrete in that section has been placed. In freezing weather, they must remain until the concrete has had sufficient time to become thoroughly hardened.

In dry but not freezing weather, the forms shall be drenched with water before the concrete is placed against them.

DEPOSITING. Each layer should be left somewhat rough

5.

to insure bonding with the next layer above; and if the concrete has already set, it shall be thoroughly cleaned by scrubbing with coarse brushes and water before the next layer is placed upon it.

Concrete shall be deposited in the moulds in layers of such thickness and position as shall be specified by the engineer in charge. Temporary planking shall be placed at the ends of partial layers, so that none shall run out to a thin edge. In general, excepting in arch work, all concrete must be deposited in horizontal layers of uniform thickness throughout.

The work shall be carried up in sections of convenient length and the sections shall be completed without intermission.

In no case shall work on a section stop within eighteen inches of the top.

Concrete shall be placed immediately after mixing, and any having initial set shall be rejected.

FACING. The facing shall be made by carefully working the coarse stone back from the form by means of a shovel, so as to bring the excess mortar of the concrete.

About one inch of mortar (not grout) of the same proportion as used in the concrete may be placed next to the forms immediately in advance of the concrete, in order to secure a perfect face.

Care must be taken to remove from the inside of the forms any dry mortar in order to secure a perfect face.

PROPORTIONS. The proportions of the materials in the concrete shall be as specifically called for by the contract, or as set forth herein, upon the lines left for that purpose, the volume of cement to be based upon the actual cubic contents of one barrel of specified weight.

Structure	Parts by Volume			
	Cement	Sand	Gravel	Broken Stone
Foundation	1	3	0	6
Arch Ring	1	2	0	4
Parapet	1	2	0	5

FINISHING. After the forms are removed, which should generally be as soon as possible after the concrete is sufficiently hardened, any small cavities or openings in the face shall be neatly filled with mortar, if necessary in the opinion of the engineer. Any ridges due to cracks or joints in the lumber shall be rubbed down with a chisel or wooden float. The entire face may then be washed with a thin grout of the consistency of whitewash, mixed in the same proportions as the mortar of the concrete. The wash shall be applied with a brush. The earlier the above operations are performed the better will be the result.

WATERPROOFING. Where waterproofing is required, a thin coat of mortar or grout shall be applied for the finishing coat, upon which shall be placed a covering of suitable waterproofing material.

FREEZING WEATHER. Ordinarily concrete to be left above the surface of the ground shall not be constructed in freezing weather, Portland-cement concrete may be built under these conditions by special instructions. In this case the sand, water and broken stone shall be heated; and in severe cold, salt shall be added in the proportion of two pounds per cubic yard.

DESIGN.

THICKNESS OF ARCH AT THE CROWN.

American Practice.

$$\begin{aligned} d &= \frac{1}{4} \sqrt{p + \frac{1}{2} s} + 0.2 \\ &= \frac{1}{4} \sqrt{37.5 + 15} + 0.2 \\ &= 2.01 \text{ ft.} \end{aligned}$$

English Practice.

$$\begin{aligned} d &= \sqrt{0.12 p} \\ &= \sqrt{0.12 \times 37.5} \\ &= 2.12 \text{ ft.} \end{aligned}$$

French Practice.

$$\begin{aligned}d &= 1\frac{1}{2} + \frac{1}{23} s && \text{Feronet's Formula.} \\&= 1.083 + 1.305 \\&= 2.388 \text{ ft.}\end{aligned}$$

$$\begin{aligned}d &= 0.50 + 0.28 \sqrt{2p} && \text{Croizette-Desnoyers.} \\&= 0.50 + 0.28 \sqrt{75.0} \\&= 2.93 \text{ ft.}\end{aligned}$$

$$\text{Mean of the above, } \frac{2.01 + 2.12 + 2.88 + 2.93}{4} = 2.86 \text{ ft.}$$

Thickness used will be thirty inches.

THICKNESS OF ABUTMENT AT SPRINGING LINE.

$$\begin{aligned}\text{Trautwine.} \quad t &= 0.2 p + 0.1 r + 2.0 \\&= 7.5 + 0.6 + 2.0 \\&= 10.10 \text{ ft.}\end{aligned}$$

German.

$$\begin{aligned}t &= 1.0 + 0.04 (5s + 4n) \\&= 7.96 \text{ ft.}\end{aligned}$$

Rankine

$$\begin{aligned}t &= \frac{p}{3} \\&= 12.5 \text{ ft.} \\t &= \frac{p}{4} \\&= 9.5 \text{ ft.}\end{aligned}$$

$$\text{Mean of the above, } \frac{10.1 + 7.96 + 12.5 + 9.5}{4} = 9.5 \text{ ft.}$$

Thickness used will be 8.0 feet.

CROWN THRUST.

Navier's Principle.

$$\begin{aligned}T &= pp * \\&= (2.6 \times 150 + 3.9 \times 100 + 100) 37.5 \\&= 33,000 \text{ pounds.}\end{aligned}$$

* p = the normal pressure per unit length of intrados.

p = radius of curvature and is equal to 37.5 feet.

The thickness of the arch at the crown is taken by averaging the results from using Trautwine's, Rankine's and Perronnet's formulas; and the thickness of the abutment is the average of Trautwine's, Rankine's, and the German formula. These formulae are given in Baker's Masonry Construction. The results are quite high; and gives a massive arch which is probably needlessly heavy, but the extra material adds but little expense, and is worth its cost for insurance, particularly against future increase of the live load.

The method of loading and the calculating the stresses is taken from Baker's Masonry Construction. One half of the span is loaded with live load. As there is enough earth between the top of the crown and the street surface, the live load is assumed to be uniformly spread over the surface of the loaded half. The dead load is composed of the weight of the arch and the earth covering. For the results see pages 9 to 15 and plates 1 to 3.

The stresses as calculated are quite small, due to the thickness of the arch. No reinforcements is necessary. All the conditions for the stability and safety of the arch are satisfied, the line of resistance is found to stay well within the middle third. The temperature stresses are quite low, and it is highly improbable that the arch will be stressed to that extent, since the arch is situated in a little valley well protected from the heat and cold.

Very little attention will be given to the footings, the thickness of the abutments being sufficiently large enough to distribute the pressure and keep it within the allowable limit. The character of the soil is such that it will easily support two and one-half tons per square foot. On account of the high embankments on both sides of the creek, enough horizontal pressure can be had to counteract the outward thrust of the arch ring.

The parapet walls are as simple as they can be made. It would not pay nor be appropriate to go to great expense to produce a more ornate parapet wall.

TABLE NO. 1.

DATA FOR MAKING $\Delta S \div I$ CONSTANT.

Ref. No.	Rectified Distance from Crown. Ft.	ΔS Ft.	Depth of Arch Ring on Radius of Neutral Line at Middle of ΔS .	Cube of Depth	$I = \frac{1}{12} bd^3$	$\frac{\Delta S}{I}$
1	0.0	0.0	2.5	15.62		
2	1.2	1.2	2.50	15.62	1.30	0.924
3	2.52	1.32	2.54	16.38	1.365	0.965
4	3.90	1.38	2.58	17.17	1.431	0.965
5	5.36	1.46	2.65	18.60	1.549	0.945
6	6.90	1.54	2.70	19.68	1.640	0.940
7	8.80	1.90	2.85	23.15	2.010	0.945
8	11.30	2.50	3.15	31.25	2.610	0.955
9	20.00	3.70	4.75	107.17	8.941	0.974

TABLE NO. 2.

DATA FOR FINDING THE TRUE EQUILIBRIUM POLYGON.

Points.	Neutral Line of Arch Ring		Trial Equilibrium Polygon				True Equilibrium Polygon.				Σaky Σbmy
	Coordinates from Center Span Line		Intercepts.		Products.		Intercepts.		Products.		
	x	y	bv	$a_1 f^1 v_1 = E$	bv*x	f*x	bm	ak	bm*x	ak*y	
1	-18.6	00	00	6.06	-00	-112.8	+6.25	+4.56	x00	00	+3.93
2	-16.3	2.8	3.22	5.00	-49.2	-76.5	+3.90	+1.77	+8.4	+4.95	+1.88
3	-10.00	4.7	6.37	3.26	-63.7	-32.6	-2.20	-.19	-.94	-.89	-.12
4	-7.92	5.17	7.12	2.58	-56.3	-20.4	-.94	-.60	-4.86	-3.10	-.59
5	-6.20	5.48	7.50	2.00	-46.5	-12.4	-1.39	-.89	-7.62	-4.89	-.87
6	-4.75	5.65	7.70	1.58	-36.6	-7.5	-1.60	-1.08	-9.04	-6.09	-1.00
7	-3.30	5.79	7.81	1.11	-25.7	-3.6	-1.76	-1.18	-10.03	-6.81	-1.11
8	-1.95	5.83	7.90	.60	-15.4	-1.17	-1.82	-1.20	-10.61	-7.00	-1.15
9	-.60	5.85	7.95	.21	-4.7	-.28	-1.81	-1.29	-10.54	-7.55	-1.14
10	+.60	5.85	7.93		+4.7		-1.80	-1.29	-10.53	-7.55	-1.13
11	+1.95	5.83	7.73		+15.1		-1.74	-1.20	-10.16	-7.00	-1.09
12	+3.30	5.79	7.61		+25.0		-1.60	-1.18	-9.26	-6.81	-1.00
13	+4.75	5.65	7.42		+35.1		-1.41	-1.08	-7.96	-6.09	-.89
14	+6.20	5.48	7.18		+44.5		-1.20	-.89	-6.57	-4.89	-.75
15	+7.92	5.17	6.74		+53.4		-.78	-.60	-4.04	-3.10	-.49
16	+10.00	4.70	6.00		+60.0		00	-.19	00	-.89	-00
17	+16.3	2.80	3.00		+45.9		+2.86	+1.77	+1.42	+4.95	+1.80
18	+18.6	00	00		0		+5.88	+4.56	00	00	+3.70
		88.32	109.23		-16.4	-267.25	-0.01	-0.04	-99.90	-62.76	-0.02

R_1 acts to the left an amount equal to $-\frac{15.4}{109.28} = -0.15$ feet.

$$\bar{X}_1 = -\frac{267.25}{54.61} = -4.89 \text{ feet}$$

$$\bar{X}_2 = +4.89 \text{ feet.}$$

$$\frac{\text{True } T}{\text{trial } T} = \frac{-\bar{X}_1 - \bar{X}}{\bar{X}_1} = \frac{-4.89 + 0.15}{4.89} = 0.97$$

$$\frac{\text{True } T_1}{\text{trial } T} = \frac{\bar{X}_1 + \bar{X}}{\bar{X}_1} = \frac{4.89 + 0.15}{4.89} = 1.06$$

$$T_{1S1} = \frac{\text{true } T_1}{\text{trial } T} v_{1S1} = 1.06 \times 6.06 = 6.25$$

$$v_{1S1S} = \frac{\text{true } T_r}{\text{trial } T} v_{1S1S} = 0.97 \times 6.06 = 5.88$$

$$\text{True pole distance} = \text{trial pole distance} \times \frac{\sum my}{\sum ky}$$

$$= 33,000 \times \frac{-9990}{-62.76}$$

$$= 52,500 \text{ lbs.}$$

TABLE NO. 3.

DEAD AND LIVE LOAD STRESSES.

+signifies compression; - signifies tension.

Points.	ac	d	Stress due to:				Maximum Stress at		Max. Shear
			Bending		Thrust		Intrados	Extrados	
			lb. per sq. ft.	lb. per sq. in.	lb. per sq. ft.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	
a ₁	-.634.15	± 11540	± 80.6	+ 14750	+102.5	+ 21.9	+133.1	0.37	
a ₂	+.113.15	± 3500	± 24.3	+ 18400	+128.0	+ 83.7	+142.3	0.11	
a ₃	-.072.85	± 2820	± 19.6	+ 19100	+132.7	+113.1	+152.3	0.051	
a ₄	+.012.70	± 434	± 3.01	+ 19800	+137.5	+140.5	+134.4	0.037	
a ₅	-.022.65	± 900	± 6.25	+ 20100	+139.8	+146.0	+133.6	0.098	
a ₆	-.082.58	± 3780	± 26.3	+ 20600	+143.0	+169.3	+117.0	0.505	
a ₇	-.072.54	± 3420	± 23.7	+ 20900	+145.2	+168.9	+121.5	0.512	
a ₈	-.052.5	± 2500	± 17.5	+ 21100	+146.5	+164.2	+129.0	0.000	
a ₉	-.152.5	± 7650	± 53.5	+ 21300	+147.8	+200.3	+ 95.3	0.08	
a ₁₀	-.162.5	± 8060	± 56.0	+ 21300	+147.8	+203.8	+ 91.8	0.04	
a ₁₁	-.112.5	± 8550	± 59.4	+ 21100	+146.5	+205.9	+ 87.1	0.000	
a ₁₂	-.182.54	± 8810	± 61.2	+ 20740	+144.0	+205.2	+ 83.0	0.000	
a ₁₃	-.192.58	± 9040	± 63.0	+ 20400	+141.9	+204.6	+ 79.2	0.000	
a ₁₄	-.142.65	± 6260	± 43.4	+ 19900	+138.2	+181.6	+ 94.8	0.038	
a ₁₅	-.112.70	± 4760	± 33.0	+ 19950	+138.5	+171.5	+105.5	0.222	
a ₁₆	-.192.85	± 7400	± 51.4	+ 19650	+136.2	+ 84.8	+187.6	0.421	
a ₁₇	+.033.15	± 9500	± 65.9	+ 19650	+136.2	+ 70.3	+202.1	0.159	
a ₁₈	-.864.15	± 15700	±109.1	+ 14300	+ 99.3	- 9.8	+208.4	0.843	

TABLE NO. 4.

TEMPERATURE STRESSES.

+ signifies compression; - signifies tension.

Points Considered.	For Rise of 20° F.							For Fall of 30° F.						
	Bending		Thrust		Maximum Stress			Bending		Thrust		Maximum Stress		
					In- trados	Ex- trados	Shear					In- trados	Ex- trados	Shear
	lb. per sq. ft.	lb. per sq. in.	lb. per sq. ft.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. ft.	lb. per sq. in.	lb. per sq. ft.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.
a ₁	+17759	+123.1	+1140	+ 7.9	-115.2	+131.0	6.7	+26500	+185.0	-1710	-11.8	+173.2	-196.8	10.2
a ₂	+11950	+ 82.5	+2330	+16.1	- 66.4	+ 98.6	6.4	+17800	+123.6	-3500	-24.2	+ 99.4	-147.8	9.5
a ₃	+ 1562	+ 1.1	+3550	+24.6	+ 25.6	+ 23.6	6.2	+ 2340	+ 1.6	-5330	-36.8	- 35.2	- 38.4	9.2
a ₄	+ 5500	+ 38.2	+3860	+26.8	+ 65.0	- 11.4	4.7	+ 8240	+ 57.4	-5780	-40.3	- 97.7	+ 17.4	7.0
a ₅	+ 8450	+ 58.6	+4000	+27.8	+ 86.4	- 30.3	4.1	+12650	+ 87.9	-6000	-41.6	-129.5	+ 46.3	6.1
a ₆	+10850	+ 75.5	+4190	+29.0	+104.5	- 46.5	2.9	+16250	+113.3	-6290	-43.5	-156.8	+ 69.8	4.3
a ₇	+12500	+ 86.5	+4230	+29.3	+115.8	- 57.2	2.5	+18750	+129.7	-6350	-44.0	-173.7	+ 85.7	3.7
a ₈	+12800	+ 89.0	+4400	+30.6	+119.6	- 58.4	1.9	+19200	+133.6	-6720	-45.9	-179.5	+ 87.6	2.8
a ₉	+13300	+ 95.8	+4470	+31.6	+126.8	- 64.8	0.4	+20700	+145.1	-6720	-46.6	-191.7	+ 98.5	0.6
a ₁₀	+13800	+ 95.8	+4470	+31.6	+126.8	- 64.8	0.4	+20700	+145.1	-6720	-46.6	-191.7	+ 98.5	0.6
a ₁₁	+12800	+ 89.0	+4400	+30.6	+119.6	- 58.4	1.9	+19200	+133.6	-6720	-45.9	-179.5	+ 87.6	2.8
a ₁₂	+12500	+ 86.5	+4230	+29.3	+115.8	- 57.2	2.5	+18750	+129.7	-6350	-44.0	-173.7	+ 85.7	3.7
a ₁₃	+10850	+ 75.5	+4190	+29.0	+104.5	- 46.5	2.9	+16250	+113.3	-6290	-43.5	-156.8	+ 69.8	4.3
a ₁₄	+ 8450	+ 58.6	+4000	+27.8	+ 86.4	- 30.3	4.1	+12650	+ 87.9	-6000	-41.6	-129.5	+ 46.3	6.1
a ₁₅	+ 5500	+ 38.2	+3860	+26.8	+ 65.0	- 11.4	4.7	+ 8240	+ 57.4	-5780	-40.3	- 97.7	+ 17.4	7.0
a ₁₆	+ 1562	+ 1.1	+3550	+24.6	+ 25.6	+ 23.6	6.2	+ 2340	+ 1.6	-5330	-36.8	- 35.2	- 38.4	9.2
a ₁₇	+11950	+ 82.5	+2330	+16.1	- 66.4	+ 98.6	6.4	+17800	+123.6	-3500	-24.2	+ 99.4	-147.8	9.5
a ₁₈	+17759	+123.1	+1140	+ 7.9	-115.2	+131.0	6.7	+26500	+185.0	-1710	-11.8	+173.2	-196.8	10.2

$$Q = \frac{E I \epsilon t^\circ}{\sum \frac{B}{A} a k y} \times \frac{1}{\Delta S}$$

$$Q = \frac{1,500,000 \times 144 \times 30 \times 0.000,005,4 \times 20}{32.73 \times 1.051} = 11,150 \text{ lbs}$$

For a rise of twenty degrees the arch exerts an outward pull of 11,150 pounds. For a fall in temperature of thirty degrees Fahrenheit, the arch will exert an inward pull of $\frac{30}{20} \times 11,150 = 16,700$ pounds.

$$f_b = \frac{6 Q a \bar{k}}{d^2} = \frac{66,900 a \bar{k}}{d^2}$$

$$f_s = \frac{T}{d}$$

$$f_t = f_s + f_b = \pm \frac{T}{d} \pm \frac{66,900 a \bar{k}}{d^2}$$

$$\epsilon^1 = \frac{130.6}{1,500,000 \times 0.000,005,4} = 16.15^\circ \text{ F.}$$

Therefore the shortening of the arch ring under the action of T , the tangential component due to the dead and live load, is equal to that due to a fall of temperature of 16.15° F. ; or the bending stresses due to this shortening are 53.7% of those due to a fall of 30° , but are of opposite sign.

TABLE NO. 5.

COMBINED STRESSES DUE TO DEAD AND LIVE LOADS

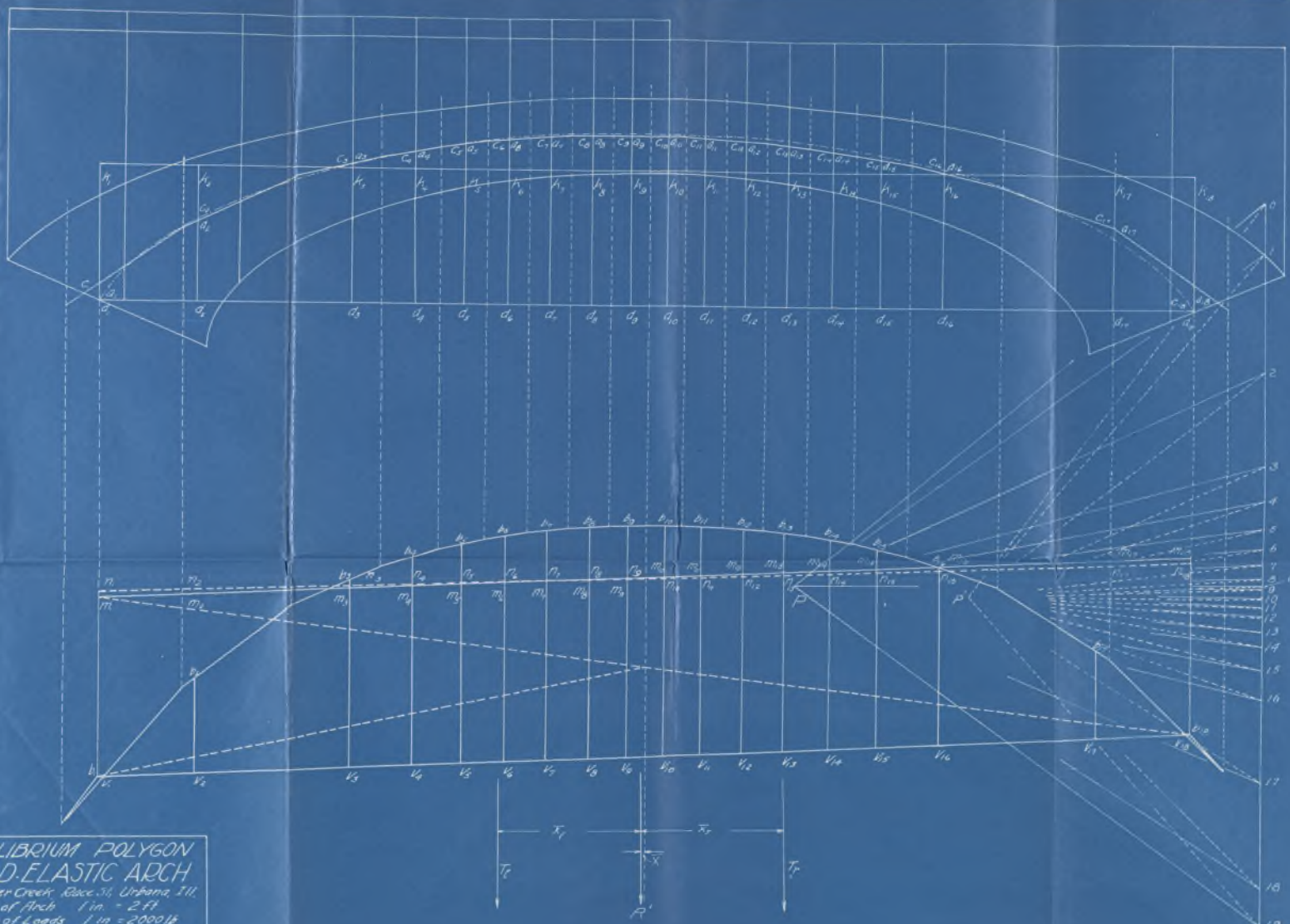
AND TO TEMPERATURE.

Results in pounds per square inch.

+ signifies compression; - signifies tension.

Points Considered.	Maximum Dead and Live Load Stresses		Bending Stresses Due to Shortening		For Rise of 20° F.				For Fall of 30° F.			
					Temperature Stresses		Combined Stresses		Temperature Stresses		Combined Stresses	
	In-trados	Ex-trados	In-trados	Ex-trados	In-trados	Ex-trados	In-trados	Ex-trados	In-trados	Ex-trados	In-trados	Ex-trados
a ₁	+ 21.9	+183.1	- 99.4	+ 99.4	-115.2	+131.0	-192.7	+413.5	+173.2	-196.8	+ 95.7	+ 85.7
a ₂	+ 83.7	+142.3	- 66.4	+ 66.4	- 66.4	+ 98.6	- 49.1	+307.3	+ 99.4	-147.8	+113.7	+ 69.9
a ₃	+113.1	+152.3	- 8.5	+ 8.5	+ 25.6	+ 23.6	+130.2	+184.4	- 35.2	- 38.4	+ 68.4	+122.4
a ₄	+140.5	+134.4	+ 30.9	- 30.9	+ 65.0	- 11.4	+236.4	+ 92.1	- 97.7	+ 17.4	+ 73.7	+120.9
a ₅	+146.0	+133.6	+ 94.4	- 94.4	+ 86.4	- 30.8	+326.8	+ 8.4	-129.5	+ 46.3	+110.9	+ 85.5
a ₆	+169.3	+117.0	+ 61.5	- 61.5	+104.5	- 46.5	+335.3	+ 9.0	-156.8	+ 69.8	+ 74.0	+125.3
a ₇	+168.9	+121.5	+ 69.7	- 69.7	+115.8	- 57.2	+354.4	- 5.4	-173.7	+ 85.7	+ 64.9	+137.5
a ₈	+164.2	+129.0	+ 71.7	- 71.7	+119.6	- 58.4	+355.5	- 1.1	-179.5	+ 87.6	+ 56.4	+144.9
a ₉	+200.3	+ 95.3	+ 78.0	- 78.0	+126.8	- 64.8	+405.1	- 37.5	-191.7	+ 98.5	+ 86.6	+115.8
a ₁₀	+203.8	+ 91.8	+ 78.9	- 78.0	+126.8	- 64.8	+498.6	- 51.0	-191.7	+ 98.5	+ 90.1	+112.3
a ₁₁	+205.9	+ 87.1	+ 71.7	- 71.7	+119.6	- 58.4	+397.2	- 43.0	-172.5	+ 87.6	+ 98.1	+103.0
a ₁₂	+205.2	+ 83.8	+ 69.7	- 69.7	+115.8	- 57.2	+390.7	- 44.1	-173.7	+ 85.7	+101.2	+ 98.8
a ₁₃	+204.6	+ 79.2	+ 61.5	- 61.5	+104.5	- 46.5	+370.6	- 28.8	-156.8	+ 69.8	+109.3	+ 87.5
a ₁₄	+181.6	+ 94.8	+ 94.4	- 94.4	+ 86.5	- 30.8	+362.5	- 30.4	-129.5	+ 46.3	+146.5	+ 46.7
a ₁₅	+171.5	+105.5	+ 30.9	- 30.9	+ 65.0	- 11.4	+267.4	+ 63.2	- 97.7	+ 17.4	+104.7	+ 92.0
a ₁₆	+ 84.8	+187.6	- 8.5	+ 8.5	+ 25.6	+ 23.6	+101.9	+219.7	- 35.2	- 38.4	+ 41.4	+157.7
a ₁₇	+ 70.3	+202.1	- 66.4	+ 66.4	- 66.4	+ 98.6	- 62.5	+367.1	+ 99.4	-147.8	+169.7	+120.7
a ₁₈	- 9.8	+208.4	- 99.4	+ 99.4	-115.2	+131.0	+224.4	+438.8	+173.2	-196.8	+ 64.0	+111.0

Total Sales	400	600	800	970	980	700	400	200
Total Costs	(130)	(160)	(170)	(200)	(200)	(150)	(100)	(70)
Total Profit	\$270	\$440	\$630	\$770	\$780	\$550	\$300	\$130



EQUILIBRIUM POLYGON
 FIXED-ELASTIC ARCH
 Over Silver Creek Race, Ill., Urbana, Ill.
 Scale of Arch 1 in. = 2 ft.
 Scale of Loads 1 in. = 2000 lb.
 Scale of Moment Arm 1 in. = 6500 lb.
 Drawn By *W. H. Hendon*
 Checked by J. P. Wiley.

